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ANSWERS

TO THE PROBLEMS IN

WENTWORTH'S COLLEGE ALGEBRA

BOSTON, U.S.A.:
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1894.

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ANSWERS

TO

EXERCISES IN THE COLLEGE ALGEBRA.

Ex. 1. Page 10.

1. $5a^2+4a+7b.$ 3. $-12a^3+7a^2b+11ab^2-11b^3.$
2. $15x^2-7xy+11y^2.$ 4. $x^5+2x^4+12x^3+9x^2-10x-12.$
5. $8m^4+10m^3n-7m^2n^2-7mn^3-9n^4.$
6. $2x^6+11x^5y-11x^4y^2+x^3y^3-3xy^5-7y^6.$

Ex. 2. Page 13.

1. $2a-4b+5c.$ 5. $5x^3+10x^2-3x-18.$
2. $5x^3-9x^2+13x-10.$ 6. $x.$
3. $a^3+8a^2b-16ab^2+12b^3.$ 7. $5b-5c.$
4. $3a^2+\frac{1}{3}ab-\frac{3}{10}b^2+\frac{1}{3}a.$ 8. $2a-4b.$
9. $-10a+15b.$
10. $2xy+xz-2yz-y^2+xz^2-x^2z.$
11. $-2x^4-15ax^3+8a^2x^2+6a^3x-5a^4.$

Ex. 3. Page 18.

1. $12x^2-7xy-10y^2.$
2. $8x^3+6x^2-20x-15.$
3. $4x^4+14x^3-2x^2-25x+12.$

4. x^8+4x^4+16.

5. $x^4-3x^3y-9x^2y^2+23xy^3-12y^4$.

6. $27x^3-y^3+18xy+8$.

7. $33a^6-a^5b-44a^2b^4-80ab^5-16b^6$.

8. $8a^3b+8ab^3$.

9. $x^2+4y^2+9z^2-4xy+6xz-12yz$.

10. $x^6+4x^5-4x^4-18x^3+12x^2+8x+1$.

11. $1170d^{3y+1}-1620d^3+1800d^{5y+2}$.

12. $600x-1050x^{m-5n+4}+625x^{2m}$.

13. $2a^{p+3}-7a^{p+2}+12a^{p+1}-19a^p+20a^{p-1}-11a^{p-2}+5a^{p-3}$.

14. $a^{3n+3}-2a^{2n+3}-2a^{2n+2}+2a^{2n+1}+a^{n+3}+2a^{n+2}-a^{n+1}$
$-2a^n+a^{n-1}$.

15. $a^{2p+2}-4a^{2p}+12a^{2p-1}-9a^{2p-2}$.

Ex. 4. Page 24.

1. $\dfrac{5}{abc^4}$.

2. $\frac{13}{2}a+4a^2x+7x+\frac{9}{2}a^2$.

3. $7x^2+16ax+4a^2$.

4. $x-2a$.

5. $9x^2+6xy+4y^2$.

6. x^2+b^2-ax.

7. $a+2b-c$.

8. $2x^2+3xy+y^2-2$.

9. a^n-a^m.

10. $125x^3+75x^2y+45xy^2+27y^3$.

11. $x^{2n}-x^ny^n+y^{2n}$.

12. $\dfrac{9a^2}{25}+\dfrac{3ab}{20}+\dfrac{b^2}{16}$.

13. $a^2+3ab+3b^2+3ac+9c^2$.

14. $a^m+2a^{m+1}-4a^{m+2}+5a^{m+3}$.

15. $4h^4-2h^3-3h^2$.

16. $2x^{m-1}+3x^{m-2}-4x^{m-3}$

Ex. 5. Pages 32, 33.

1. $x(3x+2)(3x^2+1)$.
2. $a(2a-3b)(a^2-7)$.
3. $(x+3y)(5x^2-4y^2)$.
4. $(ax-by)(ax+by)(x-c)$.
5. $(x+7)(x+1)$.
6. $(x-12)(x-5)$.
7. $(x+9)(x-2)$.
8. $(x-6)(x+4)$.
9. $(3x+5)(3x+5)$.
10. $(4x-7)(4x-7)$.
11. $(x+9)(x-8)$.
12. $(x-22)(x+8)$.
13. $x^2(9x-14y)(9x+14y)$.
14. $(3a+x)(3a-x)(9a^2-3ax+x^2)(9a^2+3ax+x^2)$.
15. $x(4x^2+y^2)(16x^4-4x^2y^2+y^4)$.
16. $x^2(x^2-2y^2)$.
17. $(a^2+ab+2b^2)(a^2-ab+2b^2)$.
18. $(x-y)(3x-5y)$.
19. $(x-1)(x-7)(3x^2+7)$.
20. $(x^2-b^2+c^2)(x^2-b^2+c^2)$.
21. $(3x-2)(5x+1)$.
22. $(x-3)(11x-21)$.
23. $(3x+5)(7x-3)$.
24. $(5x-3)(14x+3)$.
25. $(x^2+a^2-ab+b^2)(x^2-a^2-ab-b^2)$.
26. $(x^2-5)(5x^2+4x+25)$.
27. $(2x^2-x+1)(x^2-2x-2)$.
28. $(2x^2-ax+a^2)(x+a)(3x-2a)$.
29. $2(x+y)(x-y)(6x^3+5x^2y+2y^3)$.

Ex. 6. Page 40.

1. $3x-2$.
2. $x-a$.
3. x^2-4x+4.
4. $3x^2-x+4$.
5. $4x-1$.
6. $x-2$.
7. $x+3$.
8. $2a^2-3ax$.
9. a^3-2a^2+a.
10. $3x-y$.
11. $6x^3+2x^2-4x$.

12. $x^4 + 4x^3 - 13x^2 - 64x - 48.$

13. $36x^4 - 97x^2 + 36.$

14. $6x^5 + 13x^4 - 9x^3 - 6x^2 - 6x - 40.$

15. $60a^3x^6 + 190a^4x^5 + 116a^5x^4 + 64a^6x^3 + 48a^7x^2 - 30a^8x.$

16. $4x^5 + 8x^4 - 13x^3 - 26x^2 + 9x + 18.$

Ex. 7. Pages 46, 47.

1. $\frac{6a^2}{5x^2}.$ **4.** $\frac{x^2 - ax + b^2}{x^2 + ax - b^2}.$ **7.** $\frac{a^2 - x^2}{2a^2x^2}.$

2. $\frac{2x - 3}{7x - 3}.$ **5.** $\frac{3x^2 - 2}{2x^3 + 8x^2 + 12x}.$ **8.** $\frac{3x^2 - 2ax - 6a^2}{(x - a)^2}.$

3. $\frac{3c^2 - a^2}{2a^2 + c^2}.$ **6.** 0. **9.** $-\frac{2a}{(a - b)(b - c)}.$ **10.** $\frac{1}{abc}.$

11. 4. **14.** $a.$ **17.** $-\frac{4x^2(x + y)^2}{y^2(2x + y)^2}.$

12. $\frac{9bc}{4x}.$ **15.** $\frac{2x - 1}{3x + 2}.$ **18.** $\frac{1 + x^4}{x + x^3}.$

13. $\frac{x^2 - xy}{y}.$ **16.** 1. **19.** $a^4 - a^2b^2 + b^4.$

20. $\frac{8a(4a - 3x)(2a - 9x)}{27(4a + 3x)(2a + 9x)}.$

Ex. 8. Pages 52, 53.

1. 5. **2.** True for all values of x.

3. 8. **7.** $-2.$ **11.** 8.

4. 3. **8.** 5. **12.** 7.

5. 4. **9.** $\frac{3}{4}.$ **13.** 3.

6. $-1\frac{2}{11}.$ **10.** 4. **14.** 2.

15. $3\frac{5}{21}$. **16.** 2. **17.** $\frac{11}{18}$. **18.** $3\frac{3}{7}$. **19.** $-\frac{4}{11}$.

20. $\dfrac{5a^2+4a}{4a+2}$. **21.** $\dfrac{a^2bc^2+ab+c^2}{a^2-bc}$.

22. $2a-3b$. **23.** $4a$. **24.** $4a$.

Ex. 9. Pages 55, 56.

1. 12, 9.

2. 12.

3. 16.

4. $\frac{7}{12}$.

5. 16 hours.

6. $\dfrac{ac}{b-a}$ hours.

7. 25 miles per hour.

8. $23\frac{1}{3}$ days.

9. $\dfrac{ab}{a-b}$ days.

10. $\dfrac{4m^2n+2mn^2}{4m^2+4mn-n^2}$ days.

11. $2\frac{2}{9}$ miles per hour.

12. $\dfrac{ma(c-b)}{(m+1)bc}$ miles per hour.

13. 15 miles; 2 miles per hour.

14. $\dfrac{nab}{b-a}$ feet.

15. $\dfrac{ml-bl}{a-b}$ gals., $\dfrac{al-ml}{a-b}$ gals.

Ex. 10. Pages 61–63.

The upper number is the value of x, the lower that of y.

1.	6. 2.	**6.**	34. 46.	**11.**	3. 4.
2.	12. 4.	**7.**	9. 8.	**12.**	$\frac{1}{3}$. $\frac{1}{4}$.
3.	13. 3.	**8.**	12. 5.	**13.**	$4\frac{128}{286}$. $4\frac{42}{59}$.
4.	-3. -2.	**9.**	10. 5.	**14.**	-3. 122.
5.	8. 2.	**10.**	9. 7.	**15.**	$a+b$. $b-a$.

16. $\dfrac{abc\,(bc-ab-ac)}{b^2c^2-a^2b^2-a^2c^2}$. $\dfrac{abc\,(bc+ab-ac)}{b^2c^2-a^2b^2-a^2c^2}$.

17. $a+b-c$. $a-b+c$.

18. $\dfrac{a^2+ab+b^2}{a+b}$. $\dfrac{a^2-ab+b^2}{a-b}$.

19. 1. 4. 6.

20. 2. 2. 2.

21. 10. 7. 3.

22. a. b. c.

23. $\dfrac{2abc}{ac+bc-ab}$. $\dfrac{2abc}{ab+bc-ac}$. $\dfrac{2abc}{ab+ac-bc}$.

Ex. 11. Pages 63–65.

1. 17, 12.

2. $\frac{5}{7}$.

3. \$12.40, \$1.60.

4. \$900, \$400.

5. 100 lbs.

6. 75.

7. 62.

8. A, \$22,000; B, \$16,000; C, \$18,000.

9. A, \$1800; B, \$1600.

10. \$5600; \$7400.

11. From the first 10 gals.; from the second 4 gals.

12. 720 yds.; $5\frac{1}{2}$ min.; $6\frac{1}{2}$ min.

13. A in $21\frac{3}{7}$ days; B in 50 days.

14. A in $28\frac{4}{5}$ days; B in $20\frac{4}{7}$ days; C in 24 days.

15. 30 miles.

Ex. 12. Pages 68–69.

1. $16a^{12}$.

2. $27a^6x^9$.

3. $\dfrac{32a^{20}b^{10}}{243c^{15}x^{20}}$.

4. $-64b^6c^3$.

5. $a^8b^{12}c^4$.

6. $\dfrac{a}{9b}$.

7. $-125a^9b^6x^{12}$.

8. $1296a^8b^{12}c^{16}$.

9. $-\frac{9x^4}{8b^3}$. **10.** 64. **11.** $\frac{64}{27y^{14}}$.

12. $x^3+9x^2+27x+27$.

13. $1-8x+24x^2+32x^3+16x^4$.

14. $243-405x^2+270x^4-90x^6+15x^8-x^{10}$.

15. $1-24x+240x^2-1280x^3+3840x^4$
$-6144x^5+4096x^6$.

16. $1-\frac{10x}{3}+\frac{40x^2}{9}-\frac{80x^3}{27}+\frac{80x^4}{81}-\frac{32x^5}{243}$.

17. $1+3x^2+\frac{27x^4}{8}+\frac{27x^6}{16}+\frac{81x^8}{256}$.

18. $\frac{243}{32x^5}-\frac{135}{8x^3}+\frac{15}{x}-\frac{20x}{3}+\frac{40x^3}{27}-\frac{32x^5}{243}$.

19. $1+21x+189x^2+945x^3+2835x^4+5103x^5$
$+5103x^6+2187x^7$.

20. $\frac{64}{x^8}-\frac{144}{x^4}+\frac{135}{x^2}-\frac{135}{2}+\frac{1215x^2}{64}-\frac{729x^4}{256}+\frac{729x^6}{4096}$.

21. $32a^{10}b^{5m}-40a^{m+8}b^{4m+2}+20a^{2m+6}b^{3m+4}-5a^{3m+4}b^{2m+6}$
$+\frac{5}{8}a^{4m+2}b^{m+8}-\frac{1}{32}a^{5m}b^{10}$.

22. $x^{4n-8}-x^{2n-6}y^{n-4}+\frac{3y^{2n-8}}{8x^4}-\frac{y^{3n-12}}{16x^{2n+2}}+\frac{y^{4n-16}}{256x^{4n}}$.

23. $1+12x+50x^2+72x^3-21x^4-72x^5+50x^6$
$-12x^7+x^8$.

24. $1+\frac{3x}{2}-\frac{5x^3}{8}+\frac{3x^5}{32}-\frac{x^6}{64}$.

Ex. 13. Page 73.

1. $\pm 4a^4b^2$.

2. $\pm\frac{ab^2}{3}$.

3. $\pm 3a^2b^3$.

4. $\pm\frac{5}{4}$.

5. $4a^2b$.

6. $\frac{3}{ax}$.

7. $1+2x+3x^2$.
8. $3-4x-14x^2$.
9. $2-3x-x^2+5x^3$.
10. $6x^2-10a^2-a^4$.
11. $5x+3y-2$.
12. $2x^2-3y^3+4$.
13. $\frac{x^2}{2}-\frac{y^2}{3}+\frac{1}{4}$.
14. 18.0554.
15. 31.9374.
16. 1.9146.
17. 1.1454.

Ex. 14. Page 76.

1. $3-4x$.
2. x^2-x-1.
3. $a-\frac{b}{3}$.
4. $1-2x+3x^2$.
5. $3-4x-5x^2$.
6. $2x+1-\frac{5}{x}$.
7. $2x-3a+\frac{a^2}{x}$.
8. 8.026.
9. 11.786.
10. 1.481.
11. 2.755.

Ex. 15. Page 80.

5. $a^{-\frac{11}{18}}$; $b^{\frac{13}{36}}$; $c^{\frac{1}{6}}$.
6. $a^{\frac{31}{12}}$; $\frac{bc^{\frac{1}{12}}}{x^{\frac{1}{4}}}$; $a^{\frac{7}{12}}x^{\frac{1}{18}}$.
7. $64x^{\frac{3}{2}}(3a)^{\frac{1}{4}}$; $\frac{27a^3x^{\frac{9}{4}}}{8}$; $\frac{64}{27a^6x^{\frac{9}{2}}}$; $\frac{3^{\frac{1}{3}}x^{\frac{8}{3}}}{27a^4}$.
8. x^2+1.
9. $x^{4p}+x^{2p}y^{2p}+y^{4p}$.
10. $16a+8a^{\frac{6}{7}}b^{-\frac{1}{7}}+10a^{\frac{5}{7}}b^{-\frac{2}{7}}+18a^{\frac{4}{7}}b^{-\frac{3}{7}}--8a^{\frac{3}{7}}b^{-\frac{4}{7}}$
$-4a^{\frac{2}{7}}b^{-\frac{5}{7}}-5a^{\frac{1}{7}}b^{-\frac{6}{7}}-9b^{-1}$.
11. $x^{4n}-x^{3n}y^n+x^{2n}y^{2n}-x^ny^{3n}+y^{4n}$.
12. $x^{\frac{1}{4}}-y^{-1}$.
13. $a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}-a^{\frac{1}{3}}c^{-\frac{1}{6}}+b^{\frac{2}{3}}-b^{\frac{1}{3}}c^{-\frac{1}{6}}+c^{-\frac{1}{3}}$.

Ex. 16. Pages 83, 84.

5. $\sqrt{\frac{xz}{y}}$; $\frac{a^2b^3\sqrt[3]{a^2}}{c}$; $2a^3b^4x\sqrt[3]{b^2}$.

8. $6\sqrt{7} > 5\sqrt{10} > 9\sqrt{3}$.

9. $3\sqrt[3]{5} < 4\sqrt[3]{4} < 5\sqrt[3]{3}$.

10. $24\sqrt{3}$; 2.

11. $\frac{2\sqrt{3}}{9}$; $\frac{4\sqrt{15}}{3}$.

12. $4\frac{2}{3}$.

13. $3\sqrt{2} > \frac{5}{2}\sqrt[4]{4} > 2\sqrt[8]{3}$

14. $3\sqrt{19} > 5\sqrt[3]{2} > 3\sqrt[3]{3}$.

15. $\sqrt[20]{a^{18}x^9y^{19}}$; $\frac{3\sqrt[6]{2a^5b}}{a^2}$.

16. $\frac{1}{80}$; $a^3b^7\sqrt[7]{b^2}$.

Ex. 17. Page 86.

1. $29\sqrt{3}$; $33\sqrt[3]{2}$.

2. $\frac{8}{3}\sqrt{3}$; $\frac{22}{15}\sqrt{15}$.

3. $\left(\frac{a^2}{b^2}-\frac{ac}{bd}-\frac{ad}{bm}\right)\sqrt{bc}$; $\frac{8}{5}\sqrt{10}$.

4. 0.

5. $2\sqrt[3]{2}$; $3\sqrt[3]{3}$; $4\sqrt[4]{2}$; $2\sqrt[3]{2}$.

6. a^{-4}; $x^{-\frac{2}{3}}$; $p^{\frac{1}{2}}$; $a^{\frac{15}{16}}$.

7. $x^{2m}+3x^my^n+y^{2n}$.

8. $1+2x^{-\frac{1}{3}}-3x^{-\frac{2}{3}}+4x^{-1}$.

9. $x^{\frac{1}{6}}$.

10. $\dfrac{a^{\frac{2}{5}}b^{\frac{3}{4}}}{c}$.

11. $\dfrac{2b^2}{25a^{\frac{1}{4}}c^2}$.

12. $\dfrac{5\times 5^{\frac{4}{5}}a^{\frac{22}{15}}}{2b^{\frac{15}{4}}}$.

13. x^{pq}.

14. $x^{pq+p}-y^{pq-q}$.

15. 0.614.....; 3.461.....; 1.071.....; 59.394......

16. 1.154.....; 1.216.....; 3.576.....; 1.449......

Ex. 18. Pages 89, 90.

1. $\pm\sqrt{3}$.

2. $\pm\frac{1}{2}$.

3. $\pm\frac{1}{2}$.

4. $\pm\sqrt{11}$.

5. ± 5.

6. ± 3.

7. ± 5.

8. ± 2.

9. $\pm\sqrt{-\dfrac{a}{1+b^2}}$.

10. $\pm\sqrt{\dfrac{c-b}{a}}$.

11. $\pm\sqrt{\dfrac{b}{a-1}}$.

12. $\pm b$.

13. $\pm a$.

14. $\pm 3a$.

15. $\pm\dfrac{(a+b)\sqrt{3}}{2}$.

Ex. 19. Pages 95–97.

1. $5;\ -3$.

2. $8;\ 6$.

3. $4;\ -3$.

4. $7;\ -4$.

5. $7;\ 6$.

6. $9;\ 12$.

7. $1\frac{1}{2};\ -2$.

8. $1\frac{1}{4};\ -3$.

9. $4;\ 2\frac{1}{8}$.

10. $\frac{3}{4};\ -5$.

11. $1\frac{1}{2};\ -1\frac{1}{3}$.

12. $\frac{2}{3};\ 1\frac{1}{5}$.

13. $\frac{1}{3};\ \frac{5}{6}$.

14. $-1\frac{2}{3},\ -1\frac{1}{6}$.

15. $14;\ -1$.

16. $6;\ -10$.

17. $5;\ -2\frac{1}{2}$.

18. $3;\ -9$.

19. $4;\ -2\frac{1}{2}$.

20. $2;\ 9$.

21. $3;\ -8$.

22. $3;\ -\frac{4}{5}$.

23. $7;\ -2$.

24. $6;\ -\frac{1}{2}$.

25. $3;\ \frac{3}{5}$.
26. $13;\ -4\frac{1}{3}$.
27. $4;\ -5$.
28. $1;\ -1\frac{1}{4}$.
29. $5;\ 1\frac{1}{5}$.
30. $5;\ -1\frac{1}{4}$.
31. $\frac{3}{11};\ \frac{2}{11}$.
32. $8;\ -1$.
33. $1;\ -2\frac{8}{23}$.

Ex. 20. Pages 98–100.

1. $3a;\ -a$.
2. $a;\ 7a$.
3. $\frac{a+b}{2};\ \frac{a-b}{2}$.
4. $6a;\ -\frac{4a}{3}$.
5. $\frac{a \pm \sqrt{a^2+4b}}{2}$.
6. $\frac{a(a+b)}{a-b};\ \frac{a(a-b)}{a+b}$.
7. $\frac{3}{2a};\ -\frac{1}{2a}$.
8. $a;\ b$.
9. $\frac{m}{n};\ \frac{n}{m}$.
10. $\frac{a}{2};\ \frac{3a}{4}$.
11. $\frac{a}{2};\ \frac{b}{2}$.
12. $2m;\ \frac{m}{2}$.
13. $\frac{-5a \pm \sqrt{25a^2-9a}}{2}$.
14. $ab \pm a\sqrt{b^2-b}$.
15. $a;\ a+b$.
16. $a;\ -b$.
17. $a;\ 3a+b$.
18. $2b;\ \frac{ab}{2a-b}$.
19. $\frac{a}{a-b};\ \frac{b}{a-b}$.
20. $4a-3b;\ 3b-a$.
21. $\frac{3(a+b)}{2};\ -\frac{a+b}{2}$.
22. $-4a;\ -\frac{3a}{2}$.
23. $a-b;\ -(a+2b)$.
24. $a+2b;\ a-2b$.
25. $\frac{a+3b}{2};\ -\frac{3a+b}{2}$.
26. $\frac{2a+3b}{2a-3b};\ \frac{2a-3b}{2a+3b}$.
27. $\frac{a+b}{a-b};\ \frac{a-b}{a+b}$.
28. $\frac{2a+b}{2a-b};\ \frac{2a-b}{2a+b}$.
29. $a-2b;\ \frac{4b^2}{a+2b}$.
30. $\frac{a-b+c}{a-b};\ -2$.

31. c; $2a-3b-c$.

32. $a+b$; a^2-b^2.

33. $2m(2m-n)$; $n(2m+n)$.

34. $m+n$; $\frac{1}{m}+\frac{1}{n}$.

35. $\frac{b-a}{2a-3b}$; $\frac{b-a}{2a+3b}$.

36. $a+4b$; $2(a-b)$.

37. $a-8b$; $2(a+3b)$.

38. $\frac{a-6b}{2}$; $-(5a+b)$.

39. $-(2a+3b)$; $\frac{4a^2+9b^2}{2a+3b}$.

40. $\frac{a-b}{a-2b}$; $-\frac{a+b}{a+2b}$.

41. 1; $\frac{2a+3b}{2a-3b}$.

42. $2b-a$; $\frac{(a+2b)^2}{a-2b}$.

43. $\frac{a-b}{2a-3b}$; $\frac{a+b}{2a+3b}$.

44. $a-b$; $9a-b+2c$.

Ex. 21. Page 103.

Integral roots are not given, as giving them would destroy the value of the problem.

1. $2\pm 2\sqrt{-1}$.

2. $1\pm\sqrt{-1}$; $3\pm\sqrt{2}$.

3. $\frac{3\pm 3\sqrt{-3}}{2}$.

4. $\pm 3\sqrt{-1}$.

5. $\frac{-7\pm\sqrt{-35}}{2}$.

6. $\pm 3\sqrt{-1}$.

8. $\pm 2\sqrt{-2}$.

9. $-1\pm\sqrt{-2}$.

11. $2\pm\sqrt{6}$.

14. $\frac{2-a\pm\sqrt{16+4a-3a^2}}{2}$.

15. $-1\pm 2\sqrt{-1}$.

16. $\frac{1\pm\sqrt{-23}}{2}$.

17. $\frac{-9\pm\sqrt{-95}}{2}$.

18. $\frac{-7\pm\sqrt{-23}}{2}$.

Ex. 22. Page 106.

11. 1; $-\frac{1}{2}$.

12. 3; $\frac{5}{7}$.

13. 4; $-\frac{2}{3}$.

14. 7; 23.

Ex. 23. Pages 108–111.

1. 14; 15.
2. 10; 11.
3. 16; 17.
4. 12; 72.
5. $\frac{6}{5}$.
6. $\frac{2}{7}$.
7. 10; $2.40.
8. $1.80; 20 days.
9. 28 days.
10. 28 acres.
11. 40 and 45; $81 and $72.
12. 75; $4.
13. 60 yards by 40 yards.
14. 25 yards by 48 yards.
15. $360.
16. 18 hours; 24 hours.
17. 6 days.
18. 5 hours; $7\frac{1}{2}$ hours.
19. 45.
20. 68.
21. 5 miles per hour;
    1 mile per hour.
22. $65 or $35.
23. $120.
24. 100; $85.
25. 22; $22.
26. 9 hours; $6\frac{3}{4}$ hours.
27. 120 miles.

Ex. 24. Pages 116–119.

The upper values are the values of x, the lower values, the corresponding values of y.

1. $5, \ 3;$
$3, \ 5.$

2. $9, \ -3;$
$-3, \ 9.$

3. $8, \ -3;$
$3, \ -8.$

4. $10, \ 6;$
$-6, -10.$

5. $6;$
$3.$

6. $5, -4\frac{1}{2};$
$-3, \ 3\frac{1}{3}.$

7. $2, \ 2\frac{1}{2};$
$3, \ 1\frac{1}{2}.$

8. $2, \ -2;$
$5, \ 7.$

9. $3, \ -7\frac{1}{2};$
$4, \ \frac{1}{2}.$

10. $8, \ 6;$
$6, \ 8.$

11. $4, \ 3\frac{1}{17};$
$-1, \ 2\frac{13}{17}.$

12. $2, \ -\frac{2}{7};$
$4, \ -2\frac{6}{7}.$

13. $5, \quad -4\frac{4}{5};$
$3, \quad -3\frac{3}{16}.$

14. $7, \quad 3\frac{4}{5};$
$6, \quad 1\frac{1}{5}.$

15. $6, \quad 9;$
$9, \quad 6.$

16. $9, \quad -60;$
$12, \quad -22\frac{1}{2}.$

17. $3;$
$-5.$

18. $5, \quad \frac{1}{5};$
$-2, \quad -\frac{2}{5}.$

19. $\pm 9, \quad \pm 5;$
$\pm 5, \quad \pm 9.$

20. $\pm 6, \quad \pm 4;$
$\mp 4, \quad \mp 6.$

21. $\pm 3, \quad \pm \frac{1}{\sqrt{2}};$
$\pm 2, \quad \mp \frac{5}{\sqrt{2}}.$

22. $\pm 3, \quad \pm \frac{7}{\sqrt{2}};$
$\pm 1, \quad \mp 4\sqrt{2}.$

23. $\pm 3, \quad \pm 4\frac{1}{2};$
$\pm 2, \quad \pm \frac{1}{2}.$

24. $\pm 6;$
$\pm 4.$

25. $\pm 4, \quad \pm 4\frac{4}{7};$
$\pm 2, \quad \pm \frac{6}{7}.$

26. $\pm 5, \quad \pm 3\sqrt{-3};$
$\mp 1, \quad \pm 2\sqrt{-3}.$

27. $\pm 5, \quad \pm 11\sqrt{-2};$
$\pm 2, \quad \mp \frac{9}{2}\sqrt{-2}.$

28. $\pm 4, \quad \pm \frac{2}{\sqrt{7}};$
$\mp 3, \quad \mp \frac{15}{\sqrt{7}}.$

29. $\pm 2, \quad \pm \sqrt{2};$
$\pm 4, \quad \pm 3\sqrt{2}.$

30. $\pm 2, \quad \pm \frac{7}{2\sqrt{2}};$
$\mp 4, \quad \pm \frac{3}{2\sqrt{2}}.$

31. $\pm 2, \quad \pm \frac{34}{\sqrt{19}};$
$\pm 4, \quad \mp \frac{40}{\sqrt{19}}.$

32. $\pm 2, \quad \pm \frac{4}{\sqrt{3}};$
$\pm 1, \quad \pm \frac{1}{\sqrt{3}}.$

33. $\pm 5, \quad \pm 4\frac{2}{7};$
$\pm 3, \quad \pm 4\frac{3}{7}.$

34. $4, \quad 1;$
$1, \quad 4.$

35. $5, \quad -3;$
$3, \quad -5.$

36. $7, \ -4;$
$-4, \ 7.$

37. $7, \ -5;$
$5, \ -7.$

38. $5, \ 3;$
$3, \ 5.$

39. $11, \ -3;$
$3, \ -11.$

40. $4, \ 3;$
$3, \ 4.$

41. $5, \ -3;$
$3, \ -5.$

42. $3, \ 6, \ \dfrac{-3 \pm \sqrt{33}}{2};$
$6, \ 3, \ \dfrac{-3 \mp \sqrt{33}}{2}.$

43. $2, \ -3, \ \dfrac{19 \pm \sqrt{-95}}{2};$
$-3, \ 2, \ \dfrac{19 \mp \sqrt{-95}}{2}.$

44. $6, \ 3;$
$3, \ 6.$

45. $4, \ -2;$
$2, \ -4.$

46. $6, \ 3;$
$3, \ 6.$

47. $3\frac{3}{5}, \ 1\frac{4}{5}, \ 0;$
$1\frac{4}{5}, \ 3\frac{3}{5}, \ 0.$

48. $5, \ 2, \ 6, \ 1;$
$2, \ 5, \ 1, \ 6.$

49. $3, \ 2, \ \dfrac{-3 \pm \sqrt{17}}{2};$
$2, \ 3, \ \dfrac{-3 \mp \sqrt{17}}{2}.$

50. $0, \ 3, \ 2;$
$0, \ 2, \ 3.$

51. $7, \ 2, \ -4 \pm \sqrt{19};$
$2, \ 7, \ -4 \mp \sqrt{19}.$

52. $4, \ -3, \ -19 \pm 6\sqrt{-3};$
$-3, \ 4, \ -19 \mp 6\sqrt{-3}.$

53. $5, \ -3, \ 1 \pm \sqrt{-22};$
$-3, \ 5, \ 1 \mp \sqrt{-22}.$

54. $3, \ -2, \ \dfrac{1 \pm 3\sqrt{-3}}{2};$
$2, \ -3, \ \dfrac{-1 \pm 3\sqrt{-3}}{2}.$

55. $5, \ 3, \ 4 \pm \sqrt{-33};$
$3, \ 5, \ 4 \mp \sqrt{-33}.$

56. $0, \ 4, \ 2, \ \dfrac{3 \pm \sqrt{-55}}{8};$
$0, \ 2, \ 4, \ \dfrac{3 \mp \sqrt{-55}}{8}.$

57. $5, \ 3, \ \dfrac{-5 \pm \sqrt{17}}{2};$
$3, \ 5, \ \dfrac{-5 \mp \sqrt{17}}{2}.$

58. $\pm 6, \ \mp 6;$
$\pm 3, \ \mp 3.$

59. $\pm 3,\ \pm 2;$
$\mp 2,\ \mp 3.$

60. $5,\ -5,\ 3,\ -3;$
$3,\ -3,\ 5,\ -5.$

61. $8,\ 2;$
$2,\ 8.$

62. $27,\ 3;$
$3,\ 27.$

63. $3,\ -3,\quad 3,\ -3,\ 2,\ -2,\quad 2,\ -2;$
$2,\ -2,\ -2,\quad 2,\ 3,\ -3,\ -3,\quad 3.$

$$\frac{1\pm\sqrt{-31}}{2},\ \frac{-1\pm\sqrt{-31}}{2},\ \frac{1\pm\sqrt{-31}}{2},\ \frac{-1\pm\sqrt{-31}}{2};$$

$$\frac{1\mp\sqrt{-31}}{2},\ \frac{-1\mp\sqrt{-31}}{2},\ \frac{-1\pm\sqrt{-31}}{2},\ \frac{1\pm\sqrt{-31}}{2}.$$

64. $3,\ -4\tfrac{1}{2};$
$-2,\ 2\tfrac{1}{2}.$

65. $\dfrac{a}{2};$
$\dfrac{b}{2}$

66. $\dfrac{a+2b}{2},\ \dfrac{a-2b}{2};$
$\dfrac{a-2b}{2},\ \dfrac{a+2b}{2}.$

67. $0,\ a+b,\ \tfrac{1}{2}[a-b\mp\sqrt{(a-b)(a+3b)}];$
$0,\ a+b,\ \tfrac{1}{2}[a-b\pm\sqrt{(a-b)(a+3b)}].$

68. $\pm\dfrac{a^2+b^2}{a-b};$
$\pm\dfrac{2ab}{a-b}.$

69. $3,\ 2,\ -3\pm\sqrt{3};$
$2,\ 3,\ -3\mp\sqrt{3}.$

70. $\pm 2,\ \pm 3\sqrt{\tfrac{2}{17}},\ \pm 4,\ \pm 6\sqrt{\tfrac{2}{17}};$
$\pm 4,\ \pm 6\sqrt{\tfrac{2}{17}},\ \pm 2,\ \pm 3\sqrt{\tfrac{2}{17}}.$

71. $1,\ 1,\ 4,\ -4;$
$4,\ -4,\ 1,\ 1.$

72. $\pm\sqrt{\dfrac{a}{2b}};$
$\pm\sqrt{\dfrac{b}{2a}}.$

73. $0,\ \frac{a}{2}(\sqrt{-1} \pm \sqrt{3}),\quad -\frac{a}{2}(\sqrt{-1} \pm \sqrt{3});$

$0,\ \frac{a}{2}(\sqrt{-1} \mp \sqrt{3}),\quad -\frac{a}{2}(\sqrt{-1} \mp \sqrt{3}).$

74. $a \pm b,\quad -a \pm b;$
$a \mp b,\quad -a \mp b.$

75. $\pm(2a+b),\quad \pm(a+2b);$
$\pm(a+2b),\quad \pm(2a+b).$

76. $0,\ \frac{a+2 \pm \sqrt{a^2-4}}{2b};$

$0,\ \frac{a+2 \mp \sqrt{a^2-4}}{2b}.$

77. $a+2b,\quad 2a+b,\quad \pm ab\sqrt{-1};$
$2a+b,\quad a+2b,\quad \mp ab\sqrt{-1}.$

78. $6,\ 1\frac{26}{48};$
$2,\ 5\frac{33}{48};$
$3,\ 3\frac{27}{48}.$

79. $3,\ -2\frac{2}{9};$
$4,\ -2\frac{26}{27};$
$4,\ -6\frac{4}{9}.$

80. 8, 2;
4, 4;
2, 8.

81. 2, -3;
4, -5;
6, -7.

82. $\frac{a^2}{a+b},\quad -\frac{a^2}{a+b};$

$\frac{2ab}{a+b},\quad -\frac{2ab}{a+b};$

$\frac{b^2}{a+b},\quad -\frac{b^2}{a+b}.$

Ex. 25. Pages 120–124.

1. 7 feet by 5 feet.

2. 75 yards by 16 yards.

3. 3120 sq. in.

4. 1008 sq. in.

5. 16 and 10; or -16 and -10.

6. $\frac{1}{2}(3 \pm \sqrt{5})$ and $\frac{1}{2}(1 \pm \sqrt{5})$.

7. 7 and -2.

8. 8 and 3.

9. 48.

10. 347.

11. $\frac{2}{3}$ and $\frac{3}{4}$.

12. $\frac{3}{2}$, $\frac{4}{3}$.

13. 73 and 37.

14. 8 feet, 10 feet.

15. 150 miles; 15 and 10 miles per day.

16. 88 yards by 55 yards.

17. 15 miles; 3, $2\frac{1}{2}$, 4 miles per hour.

18. 49,500 lbs.; 20 oz.

19. 72; or 45.

20. 5 hours; 3 hours.

21. A 36 pages, B 72 pages;
or A 9 pages, B 45 pages.

22. A 10, B 6; B works $1\frac{2}{3}$ hours;
or A 8, B 12; B works $1\frac{1}{3}$ hours.

23. 290 yards.

24. 6 and $1\frac{1}{2}$ miles per hour.

25. 13.

26. 7%; 6%.

27. \$3500 at 6%; \$1500 at 7%.

28. 374; or 132. **29.** 327. **30.** 140 miles.

Ex. 26. Page 126.

1. $1,\ -2,\ \frac{-1\pm\sqrt{-3}}{2},\ 1\pm\sqrt{-3}.$

2. $\pm 1,\ \pm 2.$

3. $2,\ -\sqrt[3]{12},\ -1\pm\sqrt{-3},\ \sqrt[3]{12}\left(\frac{1\pm\sqrt{-3}}{2}\right).$

4. $\pm 3,\ \pm\frac{1}{2}.$

5. $\pm 1,\ \pm\sqrt{-1},\ \pm\frac{1}{2},\ \pm\frac{1}{2}\sqrt{-1}.$

6. 1, $\frac{1}{2}$, and eight other values.

7. $-\sqrt[3]{2},\ -\sqrt[3]{12},\ \sqrt[3]{2}\left(\frac{1\pm\sqrt{-3}}{2}\right),\ \sqrt[3]{12}\left(\frac{1\pm\sqrt{-3}}{2}\right).$

8. $0,\ -\frac{1}{2},\ \frac{1}{3},\ \frac{1\pm\sqrt{-3}}{4},\ \frac{-1\pm\sqrt{-3}}{6}.$

9. $\pm 1,\ \pm\sqrt[4]{21},\ \pm\sqrt{-1},\ \pm\sqrt[4]{21}\sqrt{-1}.$

10. $\sqrt[m]{-4},\ \sqrt[m]{1}.$

11. $\sqrt[2n]{\frac{5}{2}},\ \sqrt[2n]{-\frac{5}{6}}.$

12. $\sqrt[3n]{5},\ \sqrt[3n]{-8}.$

13. $\sqrt[m]{2a},\ \sqrt[m]{-4a}.$

14. $\pm\frac{1}{\sqrt{6}},\ \pm\frac{1}{\sqrt{-2}}.$

15. $\frac{1}{\sqrt[3]{4}},\ -\frac{1}{\sqrt[3]{9}},\ \frac{-1\pm\sqrt{-3}}{2\sqrt[3]{4}},\ \frac{1\pm\sqrt{-3}}{2\sqrt[3]{9}}.$

16. $\pm\sqrt[4]{\frac{1}{14}},\ \pm\sqrt[4]{-\frac{1}{11}}$; four other values.

17. $\pm\frac{3}{5}\sqrt{5};\ \pm\sqrt{-1}.$

18. $16;\ (-\frac{5}{4})^4.$

19. $729;\ (-\frac{27}{8})^2.$

20. $\sqrt[5]{81};\ 2\sqrt[5]{8}.$

21. $121;\ (-2)^2.$

22. $512;\ -64.$

23. $5\sqrt[3]{5};\ \frac{7}{2}\sqrt[3]{\frac{7}{2}}.$

24. $64;\ (-\frac{2}{3})^6.$

25. $81;\ (-\frac{3}{4})^4.$

26. $\frac{1}{64};\ (-\frac{3}{10})^6.$

27. $\frac{1}{125};\ -\frac{8}{729}.$

28. $\frac{64}{729};\ -\frac{1}{27}.$

29. $256;\ -\frac{729}{2}.$

30. $\frac{1}{8};\ \frac{1}{2}.$

Ex. 27. Pages 129–131.

1. $5;\ 221.$

2. $5;\ 1\frac{24}{121}.$

3. $\frac{25}{36}.$

4. $\pm 5;\ \pm 3\sqrt{2}.$

5. $0;\ -28.$

6. $3;\ 12.$

7. $1;\ \frac{1}{21}.$

8. $9;\ -7.$

9. $-4;\ \frac{36}{17}.$

10. $-2;\ \frac{42}{13}.$

11. $\frac{1}{2};\ -2;\ 1\frac{1}{2};\ -3.$

12. $7;\ -4;\ 4;\ -1.$

13. $1;\ -\frac{1}{2};\ \frac{3\pm\sqrt{41}}{12}.$

14. $1\frac{1}{3};\ 3\frac{2}{3};\ 4;\ 1.$

15. $2;\ \frac{3}{2};\ \frac{21\pm\sqrt{-71}}{12}.$

16. $1;\ -\frac{4}{3};\ \frac{-2\pm\sqrt{313}}{12}.$

17. $-1;\ 2\frac{1}{2};\ \frac{3\pm\sqrt{-15}}{4}.$

18. $4;\ 1;\ 3;\ 2.$

19. $5;\ -3;\ 1;\ 1.$

20. $1\frac{1}{3};\ -2;\ \frac{-2\pm\sqrt{-62}}{6}.$

21. $2;\ -\frac{1}{2};\ \frac{3\pm\sqrt{-39}}{4}.$

22. $1.$

23. $\pm\sqrt{5};\ \pm\sqrt{65}.$

24. $1\frac{1}{4}.$

25. $\pm 2.$

26. $0;\ 2.$

27. $\pm 2.$

28. $\frac{2}{3};\ 2\frac{8}{21}.$

29. $4;\ -\frac{1}{4}.$

30. $-2a.$

31. $2;\ \frac{1}{2}.$

32. $\pm 9\sqrt{2}.$

33. $\frac{a(1\pm\sqrt{5})}{2}.$

34. $x=\pm\sqrt{\frac{a^4-b^4}{7a^2-2b^2}}.$

35. $4.$

36. $\pm 8.$

37. $-1;\ +2;\ -1\pm\sqrt{-3},\ \frac{1}{2}\pm\frac{1}{2}\sqrt{-3}.$

38. $4\frac{1}{2}.$

39. $0;\ \pm\sqrt{3}.$

40. $\pm 3.$

41. $\frac{1\pm\sqrt{1+4b^2}}{2a}.$

42. $b\pm\sqrt{b^2-ab}.$

43. $0;\ \frac{9a}{16}.$

44. $2,\ 6,\ \frac{-9\pm\sqrt{33}}{2}.$

$6,\ 2,\ \frac{-9\mp\sqrt{33}}{2}.$

45. $\frac{1}{2}(a+b+1)\pm\frac{1}{4}(\sqrt{4a+1}\pm\sqrt{4b+1}).$

$\frac{1}{2}(a-b)\qquad\pm\frac{1}{4}(\sqrt{4a+1}\mp\sqrt{4b+1}).$

46. $\frac{1}{4b}\left[(a^2+b^2)\pm\sqrt{10a^2b^2-3a^4-3b^4}\right].$

$\frac{1}{4b}\left[(a^2+b^2)\mp\sqrt{10a^2b^2-3a^4-3b^4}\right].$

47. $\pm 4, \ \pm 8; \ \pm 9, \ \pm 18.$ **48.** $0, \ 3 \pm 2\sqrt{2}; \ 0, \ (3 \pm 2\sqrt{2})^2.$ **49.** $6, \ -4\frac{1}{2}; \ 12, \ -9.$

Ex. 28. Pages 135, 136.

1. $x^2 - 5x + 6 = 0.$ **4.** $6x^2 - 7x + 2 = 0.$

2. $x^2 + x - 20 = 0.$ **5.** $12x^2 + 13x + 3 = 0.$

3. $x^2 + 14x + 48 = 0.$ **6.** $x^2 - 2ax + a^2 - 9b^2 = 0.$

7. $9x^2 - 9(a + b)x + 2a^2 + 5ab + 2b^2 = 0.$

8. $x^2 - 4x + 1 = 0.$ **11.** $3(x - 7)(x + 2).$

9. $x^2 + 2x - 4 = 0.$ **12.** $(3x - 14)(3x + 5).$

10. $3x^2 - 6x + 1 = 0.$ **13.** $(7x + 6)(7x + 1).$

14. $(13x - 2)(13x - 2).$

15. $\left(x - \frac{3 + \sqrt{-7}}{2}\right)\left(x - \frac{3 - \sqrt{-7}}{2}\right).$

16. $\left(x + \frac{1 + \sqrt{-3}}{2}\right)\left(x + \frac{1 - \sqrt{-3}}{2}\right).$

17. $(2x - 7)(2x - 7).$

18. $(2x + 3 + 2\sqrt{-1})(2x + 3 - 2\sqrt{-1}).$

19. 17. **23.** $\frac{175}{64}.$ **27.** $\frac{961}{64}.$

20. 56. **24.** $\frac{33}{7}.$ **28.** $-\frac{p}{q}.$

21. $\frac{33}{64}.$ **25.** $\frac{175}{512}.$ **29.** $-pq.$

22. $\frac{33}{8}.$ **26.** 833. **30.** $3pq - p^3.$

31. $p^2q - 2q^2.$

32. $p^4 - 4p^2q + 2q^2.$

33. $\frac{p^4}{q^2} - \frac{4p^2}{q} + 2.$

34. When the sign of b is unlike that of a and c, and $b^2 - 4ac > 0$.
When a, b, c have like signs, and $b^2 - 4ac > 0$.
When a and c have unlike signs.

35. When $a^2c + ac^2 + b^3 = 3abc$. **36.** When $b + c = 0$.

Ex. 29. Pages 141, 142.

1. $m = 1$ or $-\frac{1}{2}$; m between 1 and $-\frac{1}{2}$; m greater than 1 or less than $-\frac{1}{2}$.

2. $m = 3$ or $\frac{5}{7}$; m between 3 and $\frac{5}{7}$; m greater than 3 or less than $\frac{5}{7}$.

3. $m = 4$ or $-\frac{2}{3}$; m between 4 and $-\frac{2}{3}$; m greater than 4 or less than $-\frac{2}{3}$;

4. $m = 7$ or 23; m greater than 23 or less than 7; m between 7 and 23.

5. $m = 0$ or 4; m greater than 4 or less than 0; m between 0 and 4.

6. Min. value 4.

7. Min. value 7.

8. Max. value 7.

9. Min. value 4.

10. Min. value 16.

11. Min. value -32.

12. Max. value $\frac{1}{24}$.

13. Max. value $\frac{225}{144}$.

14. Max. value $\frac{1}{2}$.

15. Max. value 1, min. value $-1\frac{2}{3}$.

16. Min. value -1, max. value $+2$.

17. No max. or min.

18. Max. value $\frac{11}{2}$, min. value $\frac{1}{2}$.

19. Max. value 4, min. value 0.

20. Max. value 3, min. value $1\frac{1}{2}$.

21. Bisect the line.

22. Bisect the line.

23. Each part is a.

24. The altitude of the rectangle is half the altitude of the triangle.

25. The inscribed square.

26. The inscribed square.

Ex. 30. Page 145.

1. $\pm(3+\sqrt{5})$.

2. $\pm(2\sqrt{3}+\sqrt{5})$.

3. $\pm(\sqrt{7}+\sqrt{3})$.

4. $\pm(\sqrt{11}+\sqrt{5})$.

5. $\pm(\sqrt{7}-\sqrt{2})$.

6. $\pm 2(\sqrt{3}-\sqrt{2})$.

7. $\pm(\sqrt{6}-\sqrt{3})$.

8. $\pm(7-3\sqrt{5})$.

9. $\pm(\sqrt{10}-\sqrt{3})$.

10. $\pm(3-\sqrt{2})$.

11. $\pm(2\sqrt{3}-\sqrt{2})$.

12. $\pm(2\sqrt{5}-3\sqrt{2})$.

13. $\pm(3\sqrt{11}-2)$.

14. $\pm(3\sqrt{5}-2\sqrt{3})$.

15. $\pm(\frac{1}{2}\sqrt{10}-1)$.

16. $\pm(\sqrt{a+b}+\sqrt{a-b})$.

17. $\pm(\sqrt{a^2-b^2}-b)$.

18. $\pm(3\sqrt{7}-2\sqrt{6})$.

19. $\pm(a-b-2\sqrt{ab})$.

Ex. 31. Pages 151, 152.

1. -4; -72; $\sqrt{15}\sqrt{-1}$.

2. $-\frac{1}{3}\sqrt{-21}$; 2; 3.

3. $4+9\sqrt{-1}$; $5+2\sqrt{6}\sqrt{-1}$; $-18+18\sqrt{3}\sqrt{-1}$.

4. 19.

5. 11.

6. $19+19\sqrt{-3}$.

7. $55-13\sqrt{-2}$.

8. $-6-26\sqrt{-15}$.

9. $\sqrt{ac}-ab\sqrt{bc}+(bc+a\sqrt{ab})\sqrt{-1}$.

10. $6-4\sqrt{-1}$; $12+2\sqrt{-7}$.

11. $\frac{3}{5}-\frac{1}{5}\sqrt{-1}$.

12. $1+6\sqrt{-2}$.

13. $\pm(2+\sqrt{-3})$.

14. $\pm(4-\sqrt{-6})$.

15. $\pm(\sqrt{3}+2\sqrt{-5})$.

16. $\pm(5-3\sqrt{-7})$.

22. $\pm(\sqrt{\frac{1}{2}}+\sqrt{-\frac{1}{2}}),\ \pm(\sqrt{\frac{1}{2}}-\sqrt{-\frac{1}{2}})$.

23. $+1,\ -1,\ \pm(\frac{1}{2}+\frac{1}{2}\sqrt{-3});\ \pm(\frac{1}{2}-\frac{1}{2}\sqrt{-3})$.

24. $\pm 1, \pm\sqrt{-1}, \pm(\sqrt{\frac{1}{2}}+\sqrt{-\frac{1}{2}}); \pm(\sqrt{\frac{1}{2}}-\sqrt{-\frac{1}{2}})$.

25. $\frac{42}{289}-\frac{13}{578}\sqrt{-1}$.

Ex. 32. Page 154.

8. The first expression.

9. The second expression.

10. The first expression.

11. The first expression.

12. The first expression.

Ex. 33. Page 162–164.

2. 10 : 9.

4. Greatest, 23 : 25 ;
least, 3 : 4.

5. The first.

6. 4 : 15.

7. 2 : 3.

8. $a+x : a+2x$.

9. $x+2 : 2x-3$.

13. 6 and 15.

14. $\frac{mq-np}{p-q}$.

16. 150.

17. $1\frac{1}{2}$.

18. $2\frac{2}{3}$.

19. $\frac{125}{1944}$.

20. $\frac{a-b}{c(a+b)}$.

21. 7.

22. $\sqrt{m^2-25}$.

31. 0.

32. $a+b,\ \frac{a-b}{2}$.

33. ± 2.

34. -1.

35. $\frac{bc-ad}{a-b-c+d}$.

Ex. 34. Pages 168, 169.

1. 1240 rods × 680 rods. **2.** 4 : 1. **3.** 5 : 9.
4. A's wages : B's wages : : 6 : 5.
5. 15 miles per hour; 12 miles per hour.
6. 8 : 7. **8.** 6 gals.
7. 2 : 1 or 1 : 2. **9.** 10 gals.
10. 45 and 30 miles per hour.
11. A, $200; B, $150. **12.** 70 units.
13. Last year the ratio was 3 : 7;
this year it is 45 : 104.

Ex. 35. Pages 174, 175.

1. $9\frac{3}{5}$. **4.** 5. **7.** $x^2y^3 = 108$.
2. $\frac{2}{9}$. **5.** $3\frac{3}{14}$. **8.** 5.
3. $41\frac{2}{3}$. **6.** $64x^2 = 9y^3$. **9.** 8.
10. $161\frac{1}{9}$ feet per second; $402\frac{7}{9}$ feet. **11.** $9\frac{1}{7}$.
12. $20\frac{4}{7}$ feet. **14.** 87.1 days. **16.** Equal.
13. $516\frac{1}{5}$ ounces. **15.** 1 inch. **17.** 10 inches.

Ex. 36. Pages 181–183.

1. 48. **2.** -9. **3.** -59. **4.** $-4\frac{1}{6}$. **5.** $-12\frac{3}{4}$.
6. 116. **7.** -55. **8.** 228.
9. $\frac{29n - 5n^2}{12}$. **13.** $d = 1\frac{1}{2}$; $l = 29\frac{1}{2}$.
14. $a = 0$; $d = 7$.
10. $\frac{59n - 5n^2}{24}$. **15.** $a = 11$; $l = 88$.
11. $d = 4\frac{1}{3}$; $s = 377$. **16.** $n = 16$; $l = 69$.
12. $d = \frac{3}{4}$; $s = 2757\frac{1}{4}$. **17.** $a = 12$; $n = 14$.

18. 717 feet per second.

19. $1\frac{11}{12}$, $2\frac{5}{6}$, $3\frac{3}{4}$, ..., $11\frac{1}{12}$.

20. 10th term.

21. 9.

22. 6 and 14.

23. 105.

24. 47.

25. $2\frac{1}{4}$, $4\frac{1}{4}$, $6\frac{1}{4}$, ...

26. 5, 7, 9.

27. 4, 11, 18.

28. -3, 1, 5, 9.

29. 10 terms.

30. 2, 6, 10, 14.

31. No; he would pay $10.24 instead of $8.

32. 284.

33. 3825 feet.

34. 3200 feet.

35. 3, 6, 9, 12, 15.

Ex. 37. Pages 188–190.

1. 384.

2. -4096.

3. $-\frac{1}{3^{19}}$.

4. $\frac{2^{17}}{3^{16}}$.

5. $(-\frac{4}{5})^{n-1}$.

6. 8188.

7. $\frac{27}{2}-\frac{1}{2\times 3^{16}}$.

8. $\frac{25}{8}-\frac{3^{12}}{8\times 5^{10}}$.

9. $\frac{75}{32}-\frac{3}{32\times 5^{n-2}}$.

10. $2\frac{2}{3}$.

11. $1\frac{1}{2}$.

12. $\frac{5}{7}$.

13. $\frac{3}{10}$.

14. $\frac{17}{111}$.

15. $\frac{1223}{9900}$.

16. $3\frac{19}{110}$.

17. $4\frac{553}{9990}$.

18. $r=\frac{1}{2}$, $s=69\frac{3}{4}$; $r=-\frac{1}{2}$, $s=24\frac{3}{4}$.

19. $a=2$, $s=254$.

20. $a=5$, $l=320$.

21. $a=1$, $n=5$.

22. 28, 56, 112.

23. 6, 18, 54, 162, 486.

24. 5th.

25. 3, 192.

26. 3, 12, 48, 192.

27. 7, 14, 28, 56.

28. 242.

29. 1, 2, 3; 7, 2, -3.

30. 2, 5, 8; 26, 5, -16.

31. 3, -9, 27, 63; 75, 15, 3, -9.

32. 1, 2, 4, 6. **33.** 3, 27. **34.** 139.

35. Any series with ratio $\frac{1}{3}$.

Ex. 38. Page 192.

1. $2\frac{2}{5}$, 3, 4, 6. **2.** 1, 9; $-7\frac{1}{5}$, $\frac{4}{5}$. **3.** 12.

4. $13\frac{1}{3}$, $14\frac{2}{17}$, ..., $17\frac{1}{7}$, $18\frac{6}{13}$.

5. Sixth term. **7.** 8, 2.

6. 6, $6\frac{2}{5}$, $6\frac{6}{7}$, $7\frac{5}{13}$. **8.** 104, 234.

9. 4, 7, 28; -5, -11, 55. **11.** $A > H$.

Ex. 39. Pages 196–198.

1.

x	y
12	0
11	1
....	....
0	12

2.

x	y
36	1
25	3
14	5
3	7

3.

x	y
11	1
2	5

4.

x	y
8	2
3	10

5.

x	y
18	5
15	10
....	....
0	35

6.

x	y
116	1
96	4
....	....
16	16

7.

x	y
36	0
35	3
....	....
0	108

8.

x	y
125	4
110	12
....	....
5	68

9. $x = 2$
$y = 1$

10. $x = 4$
$y = 3$

11. $x = 7$,
$y = 1$.

12. $x = 8$,
$y = 3$.

13. $x = 7$,
$y = 2$.

14. $x = 11$,
$y = 11$.

15.

150, 5;
133, 12;
....
14, 61.

16.

14, 6;
31, 14;
48, 22;
....

17.

6, 2;
21, 9;
36, 16;
....

18.

81, 8;
57, 32;
33, 56;
9, 80.

19.

275, 39;
132, 182.

20. 39.

21.

21, 172;
109, 260;
....

22.

$\frac{16}{24}$, $\frac{2}{16}$;
$\frac{13}{24}$, $\frac{4}{16}$;
....
$\frac{1}{24}$, $\frac{12}{16}$.

23.

\$5, \$2.
1, 41;
3, 36;
....
17, 1.

24.

Calves.	*Pigs.*
3,	33;
6,	28;
....	....
21,	3.

25. 3 pigs, 21 geese, 16 chickens;
or 6 pigs, 2 geese, 32 chickens.

26.

x	y
$5\frac{5}{13}$	$5\frac{5}{13}$
$8\frac{3}{4}$	$17\frac{1}{2}$
$23\frac{1}{3}$	70
....	....

27.

x	y
$\frac{1}{4}$	$1\frac{3}{4}$

28.

6, 56, 8;
12, 42, 16;
18, 28, 24;
....
24, 14, 32.

29. 3 ways (or 4 ways).

30. 5 ways.

31. $42c+5$.

32. $504c-1$.

33. 19 oxen, 1 sheep, 80 hens.

34. Calves \$7, lambs \$4, pigs \$3.

Ex. 40. Pages 205, 206.

1. $1+15x+90x^2+270x^3+405x^4+243x^5$.

2. $1+\frac{8x}{3}+\frac{8x^2}{3}+\frac{32x^3}{27}+\frac{16x^4}{81}$.

3. $1-\frac{4x^{\frac{3}{2}}}{3}+\frac{2x^3}{3}-\frac{4x^{\frac{9}{2}}}{27}+\frac{x^6}{81}$.

4. $64+192x^2+240x^4+160x^6+60x^8+12x^{10}+x^{12}$.

5. $\frac{32}{x^5}-\frac{20}{x^2}+5x-\frac{5x^4}{8}+\frac{5x^7}{128}-\frac{x^{10}}{1024}$.

6. $\frac{32a^5}{x^5}-\frac{20a^2}{x^2}+\frac{5x}{2a}-\frac{5x^4}{8a^4}+\frac{5x^7}{128a^7}-\frac{x^{10}}{1024a^{10}}$.

7. $729x^6-2916x^5y+4860x^4y^2-4320x^3y^3+2160x^2y^4$
$-576xy^5+64y^6$.

8. $\frac{32x^{10}}{y^5}-\frac{20x^8}{y^{\frac{10}{3}}}+\frac{5x^6}{y^{\frac{5}{3}}}-\frac{5x^4}{8}+\frac{5x^2y^{\frac{5}{3}}}{128}-\frac{y^{\frac{10}{3}}}{1024}$.

9. $\frac{a^6}{b^4}-\frac{a^{\frac{7}{2}}}{b^{\frac{9}{4}}}+\frac{3a}{8b^{\frac{1}{2}}}-\frac{b^{\frac{5}{4}}}{16a^{\frac{3}{2}}}+\frac{b^3}{256a^4}$.

10. $1+16x+108x^2+400x^3+886x^4+1200x^5+972x^6$
$+432x^7+81x^8$.

11. $a^6-3a^5x-3a^4x^2+11a^3x^3+6a^2x^4-12ax^5-8x^6$.

12. $7x^2$.

13. $-\frac{15}{256x^{14}}$.

14. $-\frac{91x^{\frac{5}{2}}}{4^{10}}$.

15. $-\dfrac{8855 \times 2^{19}}{3^{19}\sqrt[4]{x^3}}.$ **16.** $-\dfrac{595}{2048\sqrt[6]{x^{23}}}.$

17. $\dfrac{8 \times 7 \times 6 \cdots\cdots (9-r)}{1 \times 2 \times 3 \cdots\cdots r}\sqrt[3]{\dfrac{3^r}{2^r}}\sqrt[6]{x^{24-5r}}.$

18. $(-1)^r\dfrac{10 \times 9 \times 8 \cdots\cdots (11-r)}{1 \times 2 \times 3 \cdots\cdots r}\,\dfrac{x^{r-5}}{3^{\frac{10-r}{2}}2^r}.$

19. $(-1)^{r+2}\dfrac{12 \times 11 \times 10 \cdots\cdots (11-r)}{1 \times 2 \times 3 \cdots\cdots (r+2)}\,\dfrac{x^{\frac{18-3r}{2}}y^{2r-8}}{3^{\frac{r+2}{2}}2^{10-r}}.$

20. $\dfrac{168399\,x^3}{8192}.$

21. $\dfrac{173745\sqrt{3a^9}}{2048\sqrt{x}};\quad \dfrac{521235\,a^3\sqrt{x}}{2048\sqrt{2}}.$

22. $(-1)^r\dfrac{11 \times 10 \times 9 \cdots\cdots (13-r)}{1 \times 2 \times 3 \cdots\cdots (r-1)}\,\dfrac{x^{\frac{104-5r}{6}}}{2^{\frac{3r+8}{2}}}.$

25. $x^4 - \dfrac{3}{2} + \dfrac{1}{16x^4} + \left(2x^2 - \dfrac{1}{2x^2}\right)\sqrt{-1};$

$$\left(1 - \tfrac{5}{4}x^{\frac{1}{3}} + \tfrac{5}{8}x^{\frac{2}{3}} - \tfrac{5}{32}x + \tfrac{5}{256}x^{\frac{4}{3}} - \frac{x^{\frac{5}{3}}}{1024}\right)\sqrt{-1};$$

$$\left(-\frac{a^{\frac{5}{2}}}{32} - \frac{5a}{16} + \frac{5}{4a^{\frac{1}{2}}} - \frac{5}{2a^2} + \frac{5}{2a^{\frac{7}{2}}} - \frac{1}{a^5}\right)\sqrt{-1}.$$

Ex. 41. Pages 213, 214.

1. $1 + \dfrac{x}{2} - \dfrac{x^2}{8} + \dfrac{x^3}{16} - \cdots\cdots.$ **2.** $1 + \dfrac{x}{4} - \dfrac{3x^2}{32} + \dfrac{7x^3}{128} - \cdots\cdots.$

3. $1 + \dfrac{x}{3} + \dfrac{2x^2}{9} + \dfrac{14x^3}{81} + \cdots\cdots.$

4. $1 + 4x + 10x^2 + 20x^3 + \cdots\cdots.$

5. $1+\frac{5x}{2}+\frac{15x^2}{8}+\frac{5x^3}{16}+\cdots\cdots.$

6. $\frac{1}{a^{\frac{1}{2}}}+\frac{x^2}{4a^{\frac{5}{2}}}+\frac{5x^4}{32a^{\frac{9}{2}}}+\frac{15x^6}{128a^{\frac{13}{2}}}+\cdots\cdots.$

7. $2^{\frac{1}{5}}-\frac{3\times 2^{\frac{1}{5}}}{10}x-\frac{9\times 2^{\frac{1}{5}}}{50}x^2-\frac{81\times 2^{\frac{1}{5}}}{500}x^3-\cdots\cdots.$

8. $2^{\frac{2}{3}}\left(1-\frac{x^2}{3}-\frac{x^4}{36}-\frac{x^6}{162}-\cdots\cdots\right).$

9. $1-\frac{3x^2}{2}+\frac{21x^4}{8}-\frac{77x^6}{16}+\cdots\cdots.$

10. $-\frac{33x^7}{16}.$ **11.** $\frac{559130x^9}{81a^{\frac{29}{3}}}.$

12. $(-1)^{r-1}\frac{2\times 5\times 8\cdots\cdots(3r-4)x^r}{1\times 2\times 3\cdots\cdots r\times 3^r a^{\frac{3r-1}{3}}}.$

13. $\frac{5\times 7\times 9\cdots\cdots(2r+3)\times 2^r x^{2r}}{1\times 2\times 3\cdots\cdots r\times a^{2r+5}}.$

14. 8.06225. **15.** 2.002224.

16. $1+\frac{4x}{5}-\frac{2x^2}{25}-\frac{196x^3}{125}-\cdots\cdots.$

17. 351. **19.** $-\frac{22x^5}{729}.$

20. $1+x+\frac{x^2}{2}+\frac{x^3}{2}+\frac{3x^4}{8}+\frac{3x^5}{8}+\cdots\cdots.$

23. r even :

$$\frac{1\times 3\times 5\cdots\cdots 3r-1}{2\times 4\times 6\cdots\cdots 3r}.$$

r odd :

$$-\frac{1\times 3\times 5\cdots\cdots 3r-2}{2\times 4\times 6\cdots\cdots 3r-1}.$$

Ex. 42. Pages 222, 223.

1. 0.7781.
2. 1.1761.
3. 1.3222.
4. 1.1461.
5. 1.3980.
6. 1.4771.
7. 1.6232.
8. 2.6232.
9. $\bar{2}.3222$.
10. $\bar{1}.5441$.
11. $\bar{3}.5441$.
12. $\bar{3}.6020$.
13. 0.3222.
14. 1.2040.
15. $\bar{2}.7481$.
16. $\bar{1}.7993$.
17. 0.9030.
18. 1.3980.
19. 3.3804.
20. 3.4950.
21. 0.1003.
22. 0.3495.
23. 0.1398.
24. 0.0768.
25. 0.2258.
26. 0.2415.
27. 1.1650.
28. 0.3904.
29. 2.4465.
30. 0.4730.
31. 0.5243.
32. $\bar{1}.6020$.
33. $\bar{1}.4559$.
34. $\bar{1}.7781$.
35. $\bar{1}.6320$.
36. 0.2219.
37. 0.3980.
38. 0.3680.
39. 0.5441.
40. $\bar{2}.2219$.
41. $\bar{3}.3980$.
42. $\bar{2}.1461$.
43. 1.8539.
44. 1.2219.
45. $\bar{1}.5441$.
46. 0.4559.
47. $\bar{2}.8539$.
48. $\bar{5}.1707$.
49. 4.8293.
50. 5.9333.
51. 4.1040.

Ex. 43. Page 229.

1. 1.7782.
2. 2.0043.
3. 2.9996.
4. 3.5775.
5. 3.7350.
6. 3.9581.
7. 4.8490.
8. 4.0802.
9. $\bar{3}.9943$.
10. $\bar{4}.6554$.
11. $\bar{2}.1277$.
12. $\bar{1}.2983$.
13. 17,730.
14. 4295.
15. 53,910.
16. 93.98.
17. 0.1099.
18. 0.695.

Ex. 44. Pages 230, 231.

1. 41.64.
2. 0.02989.
3. 0.3131.
4. 8.294.
5. -28.47.
6. -0.3144.
7. 0.0104.
8. $-660,600$.
9. 1.308.
10. -111.3.
11. 0.001086.
12. -5.219.
13. 2.257.
14. 15.78.
15. 1.616.
16. 7.212.
17. 0.0001174.
18. 5.487.
19. 2.381.
20. 2.155.
21. 1.293.
22. 146.8.
23. 1.938.
24. 1.568.
25. 0.002431.
26. 33.04.
27. 0.5096.
28. 1.54+.
29. 2.66+.
30. 1.65+.
31. 7.53−.
32. 2.12+.
33. 0.45+.

Ex. 45. Page 232.

1. 0.6930.
2. 1.099.
3. 4.604.
4. 3.483.
5. 2.065.
6. 0.2070.
7. 0.6935.
8. $-(2.3742)$.
9. 2.808.
10. 1.262.
11. 1.585.
12. 1.209.
13. 0.9465.
14. 0.7743.
15. 1.356.
16. 2.873.
17. $-(1.262)$.
18. 0.6523.
19. $16\sqrt{2}$.
20. $\frac{3}{4}\sqrt[3]{18}$.

Ex. 46. Pages 241, 242.

1. Between 48 and 49.
2. (1) $\dfrac{B-A}{Ar}$;

   (2) $\dfrac{\log B-\log A}{\log R}$.
3. \$0.00563.
4. $4.310\times\$A$.
5. \$444, nearly.
6. \$20,270, nearly.
7. 5.35 per cent, nearly.
8. \$34.40.

9. 22, nearly.
10. 115, nearly.
11. 5 per cent, nearly.
12. 4.2 per cent, nearly.
13. $32.30.
14. 4 per cent.
15. $7377.
16. A little over 9 years.
17. A little over 16 years.
18. $969, nearly.

Ex. 47. Pages 257–259.

1. 120.
2. 180.
3. 1008; 224.
4. 15,600.
5. 6.
6. 120.
7. 5040.
8. 30,240.
9. $\dfrac{\underline{|60}}{\underline{|8}\,\underline{|52}}$.
10. 27,405 nights;
3654 times.
11. 400,400.
12. $\dfrac{\underline{|14}\,\underline{|26}}{\underline{|6}\,\underline{|8}\,\underline{|10}\,\underline{|16}}$.
13. $\dfrac{\underline{|m}\,\underline{|n}}{\underline{|r}\,\underline{|m-r}\,\underline{|s}\,\underline{|n-s}}$.
14. 396,000.
15. $3780 \times \underline{|9}$.
16. 205,200.
17. 173,641,000.
18. 5040; 144.
19. 1080.
20. 1728.
21. 432.
22. 109,600.
23. 15.
24. 10.
25. $n = 11$; $r = 3$.
26. 52.
27. 455.

Ex. 48. Pages 265, 266.

1. 216.
2. 63.
3. 7920.
4. $\dfrac{\underline{|11}}{8}$.
5. 2520.
6. 420.
7. 252.
8. 7.
9. 66.

10. $\frac{n(n+1)(n+2)}{6}$.

12. $\underline{|m+1}-1$.

11. 325.

13. 63; 32,760.

16. $(m+1)(n+1)(p+1)-2$ divisors besides 1 and the number itself.

Ex. 49. Pages 271, 272.

1. 4 to 3.
2. $\frac{1}{11}$.
3. $\frac{1}{4}$.
4. 7 to 2.
5. 7; 5 to 1.
6. $\frac{1}{6}$.
7. $\frac{13^4\underline{|4}}{52\times 51\times 50\times 49}=\frac{1}{10}$, nearly.
8. $\frac{11}{4165}$.
9. $\frac{1}{5}$.
10. $\frac{2}{9}$.
11. $\frac{1}{4}$.
12. $\frac{1}{21}$.
13. $\frac{9}{38}$.
14. 8 to 3.
16. $\frac{1}{60}$.
17. $\frac{5}{108}$.
18. The chance of doublets is 15 times that of triplets.
19. $\frac{16}{5525}$.

Ex. 50. Pages 282–284.

1. 3 to 2 in favor of 1st.
2. 43 to 34 against C.
3. $\frac{1}{9}$.
4. $\frac{11}{36}$.
5. $\frac{1}{6}$.
6. $\frac{3}{4}$.
8. $\frac{1}{5}$.
9. $\frac{33}{1000}$.
10. $\frac{1}{60}$.
11. \$9.20.
12. \$2.98$\frac{1}{3}$.
13. \$3.
14. $\frac{671}{1296}$.
15. $\frac{3}{8}$.
16. $\frac{3781}{46656}$.
17. 11 to 3.
18. $\frac{80}{243}$.
19. $\frac{224}{729}$.
20. $\frac{11}{16}$.
21. $\frac{n}{2^n}$.
22. $\frac{\underline{|n-1}}{n^{n-1}}$.
23. $\frac{12393}{12500}$.
24. $\frac{16}{27}$.
25. 37$\frac{1}{2}$ cents; 7 to 1.
26. A loss of 25 cents.
27. A \$6.83$\frac{11}{168}$; B \$13.05$\frac{125}{168}$; C \$3.86$\frac{4}{21}$.

Ex. 51. Pages 287, 288.

1. $\frac{1}{3}$. 2. $\frac{1}{2}$. 3. $\frac{3}{5}$. 4. $\frac{15}{47}$. 5. $\frac{2}{3}$.

6. $0.75 ; $3. 7. $\frac{9999}{10000}$. 8. $\frac{11}{24}$. 9. $\frac{18}{29}$.

Ex. 52. Pages 298, 299.

1. $\frac{7}{80}$; $\frac{22}{51}$; $\frac{78}{99}$.

2. $\frac{1}{1+}\frac{1}{3+}\frac{1}{1+}\frac{1}{1+}\frac{1}{1+}\frac{1}{1+}\frac{1}{1+}\frac{1}{1+}\frac{1}{2};\ \frac{7}{9};$

$3+\frac{1}{2+}\frac{1}{1+}\frac{1}{1+}\frac{1}{1+}\frac{1}{1+}\frac{1}{3};\ \frac{27}{8};$

$1+\frac{1}{2+}\frac{1}{4+}\frac{1}{1+}\frac{1}{1+}\frac{1}{3};\ \frac{29}{20};$

$\frac{1}{2+}\frac{1}{1+}\frac{1}{1+}\frac{1}{1+}\frac{1}{3+}\frac{1}{1+}\frac{1}{4};\ \frac{3}{8};$

$2+\frac{\dot{1}}{4};\ \frac{682}{305};$

$3+\frac{\dot{1}}{3+}\frac{\dot{1}}{6};\ \frac{1257}{379};$

$9+\frac{\dot{1}}{1+}\frac{1}{3+}\frac{1}{1+}\frac{\dot{1}}{18};\ \frac{921}{94}.$

3. $\frac{1}{5+}\frac{1}{2+}\frac{1}{7+}\frac{1}{3};\ \frac{15}{82};$

$2+\frac{1}{4+}\frac{1}{6+}\frac{1}{8};\ \frac{457}{204};$

$\frac{1}{2+}\frac{1}{4+}\frac{1}{6+}\frac{1}{8+}\frac{1}{10};\ \frac{25}{56};$

$5+\frac{1}{3+}\frac{1}{1+}\frac{1}{3+}\frac{1}{5+}\frac{1}{7};\ \frac{79}{15}.$

4. $4+\frac{1}{1+}\frac{1}{1+}\frac{1}{2+}\frac{1}{1+}\frac{1}{1+}\frac{1}{8+}$ etc ;

$4+\frac{1}{1+}\frac{1}{2+}\frac{1}{4+}\frac{1}{2+}\frac{1}{1+}\frac{1}{8+}$ etc. ;

$5+\frac{1}{1+}\frac{1}{2+}\frac{1}{1+}\frac{1}{10+}$ etc. ;

$7+\frac{1}{2+}\frac{1}{2+}\frac{1}{2+}\frac{1}{14+}$ etc.

5. $\frac{127}{48}$; $\frac{117}{37}$; $\frac{244}{63}$; $\frac{268}{65}$; $\frac{140}{33}$; $\frac{161}{36}$; $\frac{55}{72}$; $\frac{319}{60}$.

6. $\frac{79}{90}$. 7. $\frac{7}{33}$. 8. $\frac{79}{96}$. 9. $\frac{99}{70}$.

10. $\frac{22}{7}$; $\frac{333}{106}$; $\frac{355}{113}$.

11. 15 and 16; 61 and 65; 76 and 81; etc.

12. $\sqrt{\frac{3}{2}}$; $\sqrt{15}$; $\frac{\sqrt{197}-13}{4}$; $\frac{\sqrt{1093}-27}{14}$.

14. Error is less than 0.00012.

15. $\frac{1}{4}$, $\frac{7}{29}$, $\frac{8}{33}$, $\frac{39}{161}$, ...

16. 1, $\frac{3}{4}$, $\frac{13}{17}$, $\frac{250}{327}$, ...

Ex. 53. Pages 304, 305.

1. 2220.
2. 12,431.
3. 334,345.
4. 17,832,126.
5. 1625.
6. 231.
7. 152.
8. 342.
9. 4444.
10. 1287.
11. 13,133.
12. 2225.
13. $22.2\dot{6}$.
14. 0.0302.
15. 0.222.
16. 20.2444.
17. Scale of 11.
18. Scale of 9.
19. Scale of 3.
21. 1, 8, 16, 64, 256 lbs.

Ex. 54. Page 312.

1. 105. 2. 3. 3. 6. 4. 42.

Ex. 55. Page 321.

1. $-\frac{3}{4}$.
2. 1.
3. 2.
4. 2.
5. -2.
6. -1.
7. 1.
8. $4+2\sqrt{2}$.
9. 0.
10. -8.
11. $\frac{1}{\sqrt{2a}}$.

Ex. 56. Page 328.

1. Convergent.
2. Divergent.
3. Divergent.
4. Divergent.
5. Divergent.
6. Divergent.

Ex. 57. Pages 333, 334.

1. 77,323.
2. 11,860.
3. 631.
4. 120.
5. 2.425.
6. 766.5.
7. 58.
8. 255.
9. $3n(n+1)(n+3)$.
10. $\frac{n(n+1)(4n-1)}{6}$.
12. 11.
13. 11,879.
14. 5610.
15. 460.
16. 1540; 2870.
17. 190.
18. 4095.
19. 1840.
20. 25.
21. 5525.
22. 4970.

Ex. 59. Page 338.

The first expression is the sum to n terms; the second, to an infinite number of terms.

1. $\frac{11}{18}-\frac{3n^2+12n+11}{3(n+1)(n+2)(n+3)};\ \frac{11}{18}$.

2. $\frac{3}{4}-\frac{2n+3}{2(n+1)(n+2)};\ \frac{3}{4}$.

3. $\frac{1}{4}-\frac{1}{4(4n+1)};\ \frac{1}{4}$.

$\frac{1}{1\times2\times3}+\frac{1}{2\times3\times4}+\ldots$

4. $\frac{3}{5}-\frac{6}{5(5n+2)}$; $\frac{3}{5}$.

5. $\frac{13}{240}-\frac{6n+13}{6(3n+5)(3n+8)}$; $\frac{13}{240}$.

6. $\frac{n}{12(n+1)}$; $\frac{1}{12}$.

7. ~~$\frac{1}{1\times4}+\frac{1}{2\times5}+\frac{1}{3\times6}$.~~

8. $\frac{1}{3\times8}+\frac{1}{6\times12}+\frac{1}{9\times16}$.

Ex. 60. Pages 340, 341.

1. $\frac{3}{x+4}+\frac{4}{x-5}$.

2. $\frac{6}{x+3}-\frac{6}{x+4}$.

3. $\frac{8}{3(x-5)}-\frac{1}{3(2x-1)}$.

4. $\frac{4}{7(x+2)}+\frac{3}{7(x-5)}$.

5. $\frac{1}{x-1}-\frac{x+2}{x^2+x+1}$.

6. $\frac{3}{4x}-\frac{1}{8(x-2)}+\frac{3}{8(x+2)}$.

7. $\frac{4}{25x}-\frac{4}{5x^2}+\frac{71}{25(x+5)}$.

8. $\frac{11}{3(x-1)}+\frac{2}{(x-1)^2}+\frac{10}{3(x+2)}$.

9. $\frac{10x-7}{3(x^2+x+1)}-\frac{4}{3(x-1)}$.

10. $\frac{5}{2x-1}-\frac{4}{3x-1}$.

11. $\frac{7}{3x-5}-\frac{5}{4x+3}$.

12. $\frac{3x}{x^2+2x-5}-\frac{1}{x-3}$.

Ex. 61. Pages 345, 346.

1. $1 + 2x + 4x^2 + 8x^3 + \cdots\cdots.$

2. $\frac{1}{2} + \frac{3x}{4} + \frac{9x^2}{8} + \frac{27x^6}{16} + \cdots\cdots.$

3. $\frac{1}{2} - \frac{x}{4} + \frac{3x^2}{8} - \frac{9x^3}{16} + \cdots\cdots.$

4. $1 - 2x + x^2 + x^3 - 2x^4 + \cdots\cdots.$

5. $5 - 7x + 12x^2 - 19x^3 + \cdots\cdots.$

6. $4x + 2x^2 - 8x^3 - 22x^4 - \cdots\cdots.$

7. $-x + 2x^2 - x^3 - x^5 + \cdots\cdots.$

8. $-\frac{1}{x^2} + \frac{1}{x} - 2 + x - 2x^2 + \cdots\cdots.$

9. $-\frac{1}{x} + 2x + x^2 - 2x^4 + \cdots\cdots.$

10. $\frac{4}{x} - \frac{8}{x^2} + \frac{16}{x^3} - \frac{32}{x^4} + \cdots\cdots.$

11. $-1 + \frac{5}{x} - \frac{15}{x^2} + \frac{45}{x^3} - \cdots\cdots.$

12. $\frac{2}{x} + \frac{1}{x^2} + \frac{5}{x^3} + \frac{16}{x^4} + \cdots\cdots.$

13. $1 + \frac{1}{x} + \frac{3}{x^2} + \frac{6}{x^3} + \cdots\cdots.$

14. $\frac{5}{x^2} + \frac{8}{x^3} + \frac{11}{x^4} + \frac{14}{x^5} + \cdots\cdots.$

15. $\frac{1}{x} + \frac{1}{x^4} + \frac{1}{x^5} + \frac{1}{x^8} + \cdots\cdots.$

16. $x = y + 2y^2 + 5y^3 + 14y^4 + \cdots\cdots.$

17. $x = y + \dfrac{y^3}{3} + \dfrac{2y^5}{15} + \cdots\cdots.$

18. $x = y - \dfrac{y^2}{2} + \dfrac{y^3}{3} - \dfrac{7y^4}{24} + \cdots\cdots.$

Ex. 62. Page 351.

1. $u_k = 3xu_{k-1} + x^2u_{k-2}$; $\dfrac{1-x}{1-3x-x^2}.$

2. $u_k = 3xu_{k-1} - x^2u_{k-2}$; $\dfrac{3-7x}{1-3x+x^2}.$

3. $\dfrac{2-3x}{1-3x+2x^2}$; $(1+2^r)x^r.$

4. $\dfrac{7-20x+7x^2}{1-2x-2x^2-3x^3}$; general terms are

$\dfrac{10\times 3^{3k}+81}{13}x^{3k}$; $\dfrac{10\times 3^{3k+1}-108}{13}x^{3k+1}$;

$\dfrac{10\times 3^{3k-1}+27}{13}x^{3k-1}.$

5. $\dfrac{1+3x}{(1-x)^2}$; $(4n+1)x^n$. 6. $\dfrac{1+6x+10x^2}{1+5x+5x^2+2x^3}.$

7. $\dfrac{3-12x+11x^2}{1-6x+11x^2-6x^3}$; $(3^r+2^r+1)x^r.$

8. $\dfrac{2n^3+3n^2+7n}{6}.$ 9. $\left[\dfrac{n(n+1)}{2}\right]^2.$

Ex. 63. Page 358.

2. $e^x.$

4. 1.3863; 1.6094; 1.7918; 1.9459.

5. 1.4427; 0.9102; 0.7213; 0.6213; 0.5581; 0.5139.

11. $1 - \dfrac{x^2}{\underline{|2}} + \dfrac{x^4}{\underline{|4}} - \dfrac{x^6}{\underline{|6}} + \cdots\cdots.$ 12. $x - \dfrac{x^3}{\underline{|3}} + \dfrac{x^5}{\underline{|5}} - \dfrac{x^7}{\underline{|7}} + \cdots\cdots.$

Ex. 64. Page 364.

3. -4.
4. -3.
5. -89.
10. $a^4 - 2a^2b^2 + b^4$.
11. 0.
12. $a^3 + b^3 + c^3 - 3abc$.

Ex. 65. Pages 370, 371.

5. 30. 6. 1. 7. 2106.
8. $(a-b)(b-c)(c-a)(a+b+c)$.
9. $(a-b)(b-c)(c-a)(ab+bc+ca)$.
10. $(a-b)(b-c)(c-a)(a^2+b^2+c^2+bc+ca+ab)$.
11. $(a+b+c)(bc+ca+ab-a^2-b^2-c^2)$.
12. $(a-b)(a-c)(a-d)(b-c)(b-d)(c-d)$
$(a+b+c+d)$.
13. $(a+b+c+d)(a+b-c-d)(a-b+c-d)$
$(a-b-c+d)$.

Ex. 66. Pages 378–380.

2. $\begin{vmatrix} 1 & 1 & 1 & 1 \\ b & e & f & g \\ c & f & h & k \\ d & g & k & l \end{vmatrix}$
3. $ab^4 + ab^3c - 2ab^2c^2 - abc^3 + ac^4$.
4. $-a^4 + a^3b + a^2b^2 - ab^3$.
5. $-3abcd$.
6. $-a^3 + 3a^2b - 3ab^2 + b^3$.
7. 9. 8. 6. 9. 377.
10. $x = 1$; $y = 2$; $z = 3$.
11. $x = 5$; $y = 1$; $z = 3$.

12. $x=1;\ y=2;\ z=-1;\ w=3.$

13. $x=2;\ y=4;\ z=-1;\ w=-3.$

14. $5x^2+9x+1=0.$

15. $1-6x+9x^2+2x^4-5x^5=0.$

16. $a^3+b^3+c^3-3abc=0.$

17. $(ab'-a'b)(bc'-b'c)=(ac'-a'c)^2.$

18. $a(a^2r+bc)^2+b(ac-b^2-a^2q)(a^2r+bc)+$
$c(ac-b^2-a^2q)^2=0.$

19. Inconsistent.

20. Consistent.

Ex. 67. Page 383.

3. $-a(a-b)(b-c)(c-d).$

5. Multiply both sides of the equation to be proved by

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Ex. 68. Page 390.

1. $x^3-x^2-3x-4;\quad -9.$

2. $x^3+3x^2+6x+20;\quad +53.$

3. $2x^3+7x^2+6x+5;\quad 0.$

4. $3x^3-9x^2+29x-93;\quad +329.$

5. $ax^2-(ah-3b)x+(ah^2-3bh+3c);$
$-ah^3+3bh^2-3ch+d.$

9. $-44.$ 11. $-1553.$

10. 2749. 12. 1.2736.

Ex. 69. Page 394.

15. $x^3 - x^2 - 44x + 84 = 0.$
16. $x^3 - 3x^2 - 10x + 24 = 0.$
17. $x^3 - 4x = 0.$
18. $x^4 - 5x^2 + 4 = 0.$
19. $x^3 - 11x^2 + 40x - 50 = 0.$
20. $4x^4 - 16x^3 + 15x^2 + 4x - 4 = 0.$
21. $x^5 + 6x^4 - 13x^3 - 78x^2 + 36x + 216 = 0.$
22. $36x^4 + 36x^3 - 37x^2 - 11x + 6 = 0.$
23. $x^4 - 10x^3 + 32x^2 - 34x + 7 = 0.$
24. $200x^3 + 15x^2 - 21x + 10 = 0.$
25. $5000x^4 + 5750x^3 - 625x^2 - 405x - 18 = 0.$
26. $x^4 - 6x^3 + 18x^2 - 30x + 25 = 0.$

Ex. 70. Page 400.

1. $x^3 - 3x^2 - 10x + 24 = 0$; $x^3 + 3x^2 - 10x - 24 = 0.$
2. 17. 4. 74. 6. -14. 8. 317. 10. 12.
3. 11. 5. 15. 7. 53. 9. 51. 11. $p^2 - 2q.$
12. $3r - pq.$ 14. $q^2 - 2pr.$
13. $-p^3 + 3pq - 3r.$ 15. $p^4 - 4p^2q + 2q^2 + 4pr.$
16. $r - pq.$ 17. $2p - \frac{q^2}{r}.$ 18. $\frac{pq}{r} - 3.$
19. $\frac{2p^2q - 4pr - 2q^2}{r - pq}.$ 20. $pq = r.$
21. $36p^3r - 8p^2q^2 - 182pqr - 36q^3 + 343r^2 = 0.$
22. $2p^3 - 9pq + 27r = 0.$
23. $27r^2 - 9pqr + 2q^3 = 0.$

Ex. 72. Page 409.

1. $2x.$
2. $3x^2.$
3. $-\frac{1}{x^2}.$
4. $-\frac{2}{x^3}.$
5. $4x^3.$
6. $-\frac{3}{x^4}.$
7. $-\frac{4}{x^5}.$
8. $2x+1.$
9. $3x^2+4x.$
10. $2x+2a.$
11. $-\frac{2x}{(x^2-3)^2}.$
12. $-\frac{2}{(x+1)^3}.$

Ex. 73. Page 411.

9. $2x+1.$
10. $3x^2-12x+11.$
11. $(x-3)(3x+5).$
12. $(x-4)(4x^2-11x).$
13. $2(x-\alpha)(x-\beta)(2x-\alpha-\beta).$
14. $3x^2-2(\alpha+\beta+\gamma)x+\beta\gamma+\gamma\alpha+\alpha\beta.$
15. $4x^3+12x^2-38x-46.$
16. $4x^3-12x^2+20x-8.$

Ex. 74. Page 414.

5. $3ax^2+6bx+3c;$
$6ax+6b;$
$6a.$

6. $4ax^3+12bx^2+12cx+4d;$
$12ax^2+24bx+12c;$
$24ax+24b;$
$24a.$

7. $3x^2-2(2\alpha+\beta)x+\alpha^2+2\alpha\beta;$
$6x-4\alpha-2\beta;$
$6.$

8. $3x^2-2(\alpha+\beta+\gamma)x+\beta\gamma+\gamma\alpha+\alpha\beta;$
$6x-2(\alpha+\beta+\gamma);$
$6.$

9. $4x^3 - 6(\alpha+\beta)x^2 + 2(\alpha^2+4\alpha\beta+\beta^2)x - 2\alpha\beta(\alpha+\beta)$;
$12x^2 - 12(\alpha+\beta)x + 2(\alpha^2+4\alpha\beta+\beta^2)$;
$24x - 12(\alpha+\beta)$;
24.

Ex. 76. Page 419.

1. $y^3 + 3y^2 + 4y - 2$.
2. $y^4 + 16y^3 + 94y^2 + 246y + 245$.
3. $3y^4 + 34y^3 + 146y^2 + 281y + 200$.
4. $2y^4 - 19y^3 + 72y^2 - 131y + 86$.
5. $2y^4 - 26y^3 + 130y^2 - 299y + 263$.

Ex. 77. Pages 426, 427.

1. $x^3 + 3x^2 + 2x + 4 = 0$.
2. $x^4 + 12x^2 + 16x - 16 = 0$.
3. $2x^4 + 9x^3 + 9x^2 + 162x - 324 = 0$.
4. $x^4 - 6x^2 - 24x - 64 = 0$.
5. $3x^5 - 16x^3 - 32x - 224 = 0$.
6. $y^3 - 2y^2 + 18y + 18 = 0$.
7. $y^3 + 10y^2 - 42y + 576 = 0$.
8. $y^4 + 5y^3 - 40y^2 + 2500y - 30{,}000 = 0$.
9. $y^5 + 3y^4 + 24y^3 - 72y^2 + 1296y - 23{,}328 = 0$.
13. $y^3 - 8y^2 + 12y + 9 = 0$.
14. $y^4 + 2y^3 - 8y^2 - 6y + 15 = 0$.
15. $y^3 + 4y^2 - 15y - 18 = 0$.
16. $y^4 + 17y^3 + 92y^2 + 172y + 96 = 0$.
17. $y^4 + 1.2y^3 + 1.54y^2 - 2.292y + 3.1981 = 0$.
18. $y^4 - 4.6y^3 + 3.56y^2 + 3.104y - 6.5424 = 0$.

19. $y^3 + 2y^2 - 11y - 25 = 0.$

20. $y^3 - 20y^2 + 100y - 547 = 0.$

21. $y^3 + 17y^2 + 75y - 29 = 0.$

Ex. 82. Page 449.

1. 1.154^+.
2. 3.134^+.
3. 4.264^+.
4. 2.833^+.
5. -4.532^+.
6. -1.064^+.
7. -0.732^+.

Ex. 83. Pages 455, 456.

1. 1.879385^+.
2. 3.263389^+.
3. 1.580947^+.
4. 2.656116^+.
5. 0.593685^+; 2.047275^+.
6. 2.381966^+; 4.618035^+.
7. 32.865383^+.
8. 624.
9. ± 29.
10. 43.
11. -2.469547^+.
12. -2.768345^+.
13. 2.356896; 2.692021
14. 2.121320^+; 2.123113^+.
15. 3.213127^+; 3.229521^+.

Ex. 84. Page 462.

1. [3, 4]; [4, 5]; [-3, -4].
2. [4, 5]. Two imaginary roots.
3. Two roots between 0 and -1;
   two roots between 2 and 3.
4. [3, 4]; [0, -1]. Two imaginary roots.

5. Roots all imaginary.

6. [0, 1]; [−2, −3]. Two imaginary roots.

7. −1, −1, [0, 1]. Two imaginary roots.

8. [0, 1]. Four imaginary roots.

Ex. 85. Page 467.

1. $\pm 1;\ \dfrac{-7 \pm 3\sqrt{5}}{2}.$

2. $\dfrac{-1+\sqrt{2} \pm \sqrt{-1-2\sqrt{2}}}{2};$

   $\dfrac{-1-\sqrt{2} \pm \sqrt{-1+2\sqrt{2}}}{2}.$

3. $\pm 1;\ \dfrac{3+\sqrt{-7} \pm \sqrt{-14+6\sqrt{-7}}}{4};$

   $\dfrac{3-\sqrt{-7} \pm \sqrt{-14-6\sqrt{-7}}}{4}.$

4. $2 \pm \sqrt{3};\ \dfrac{1 \pm \sqrt{-3}}{2}.$

5. $1;\ 1;\ \dfrac{1 \pm \sqrt{-15}}{4}.$

6. $-1;\ 2 \pm \sqrt{3};\ \dfrac{1 \pm \sqrt{-3}}{2}.$

7. $3 \pm 2\sqrt{2};\ 2 \pm \sqrt{3}.$

8. $-1;\ \dfrac{1-m \pm \sqrt{m^2-2m-3}}{2}.$

9. $-1;\ \dfrac{\sqrt{3} \pm \sqrt{-1}}{2};\ \dfrac{-\sqrt{3} \pm \sqrt{-1}}{2}.$

10. $1;\ \pm\sqrt{-1};\ \dfrac{-1 \pm \sqrt{-35}}{6}.$

Ex. 86. Page 469.

1. $\pm\sqrt{-1}\,;\ \dfrac{\sqrt{3}\pm\sqrt{-1}}{2}\,;\ \dfrac{-\sqrt{3}\pm\sqrt{-1}}{2}.$

2. $\pm 1\,;\ \pm\sqrt{-1}\,;\ \sqrt{\tfrac{1}{2}}\pm\sqrt{-\tfrac{1}{2}}\,;\ -\sqrt{\tfrac{1}{2}}\pm\sqrt{-\tfrac{1}{2}}.$

3. $1\,;\ \dfrac{-1\pm\sqrt{-3}}{2}\,;\ \sqrt[3]{\dfrac{-1\pm\sqrt{-3}}{2}}.$

4. $3\,;\ \dfrac{-3+3\sqrt{5}\pm 3\sqrt{-10-2\sqrt{5}}}{4}\,;$

$$\frac{-3-3\sqrt{5}\pm 3\sqrt{-10-2\sqrt{5}}}{4}.$$

5. $z^5+z^4-4z^3-3z^2+3z+1=0.$

Ex. 87. Page 473.

1. $-2+2\sqrt[3]{2}+\sqrt[3]{4}\,;\ -2+2\omega\sqrt[3]{2}+\omega^2\sqrt[3]{4}\,;$
$-2+2\omega^2\sqrt[3]{2}+\omega\sqrt[3]{4}.$

2. $\dfrac{2+\sqrt[3]{2}+2\sqrt[3]{4}}{3}$; etc.

3. $1+\sqrt[3]{3}+\sqrt[3]{9}$; etc.

4. $2+\dfrac{\sqrt[3]{6}+\sqrt[3]{36}}{3}$; etc.

5. $-m+m\sqrt[3]{m}+\sqrt[3]{m^2}$; etc.

Ex. 88. Page 479.

1. $3\pm\sqrt{2},\ 3\pm\sqrt{2}.$

2. $2\pm\sqrt{7},\ -2\pm\sqrt{2}.$

3. $2\pm\sqrt{7},\ 2\pm\sqrt{-3}.$

4. $2\pm\sqrt{-11},\ \dfrac{7\pm\sqrt{37}}{2}.$

5. $6\pm\sqrt{23},\ \dfrac{-5\pm\sqrt{77}}{2}.$

8. $I=0$. Reducing cubic is $4a^6\theta^3=G^2+4H^3$, and in equation (6), page 475, θ_1, θ_2, θ_3, are the three cube roots of $\dfrac{G^2+4H^3}{4a^6}$.

$J=0$. The reducing cube is $4a^3\theta^3-Ia\theta=0$, of which the roots are 0, $\pm\dfrac{I}{2a}$. Therefore

$$ax+b=\sqrt{-H}+\sqrt{\tfrac{1}{2}aI-H}+\sqrt{-\tfrac{1}{2}aI-H}.$$

10. $\theta^3-3bc\theta-(b^3+c^3)=0$.

Ex. 89. Page 494.

1. $2a^4-12a^2b^2+2b^4$.

2. $\frac{6}{5}$. **3.** 8. **6.** $\frac{5}{84}$.

7. $\dfrac{1}{2\sqrt[3]{2}}[1+\sqrt{3}+i(\sqrt{3}-1)]$;

$\dfrac{1}{2\sqrt[3]{2}}[1-\sqrt{3}-i(1+\sqrt{3})]$;

$\dfrac{1}{\sqrt[3]{2}}(-1+i)$.

8. Cf. Ex. 2, § 511.

9. $\pm\left(\sqrt{\tfrac{1}{2}\sqrt{5}+1}+i\sqrt{\tfrac{1}{2}\sqrt{5}-1}\right)$;

$\pm\left(\sqrt{\tfrac{1}{2}\sqrt{5}-1}-i\sqrt{\tfrac{1}{2}\sqrt{5}+1}\right)$.

10. 3.5844 ; − 3.3300 ; − 0.2544.

11. 1.212 : − 1.571 ; − 1.1427.

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